Comparative study of transformer robustness for multiple particle tracking without clutter

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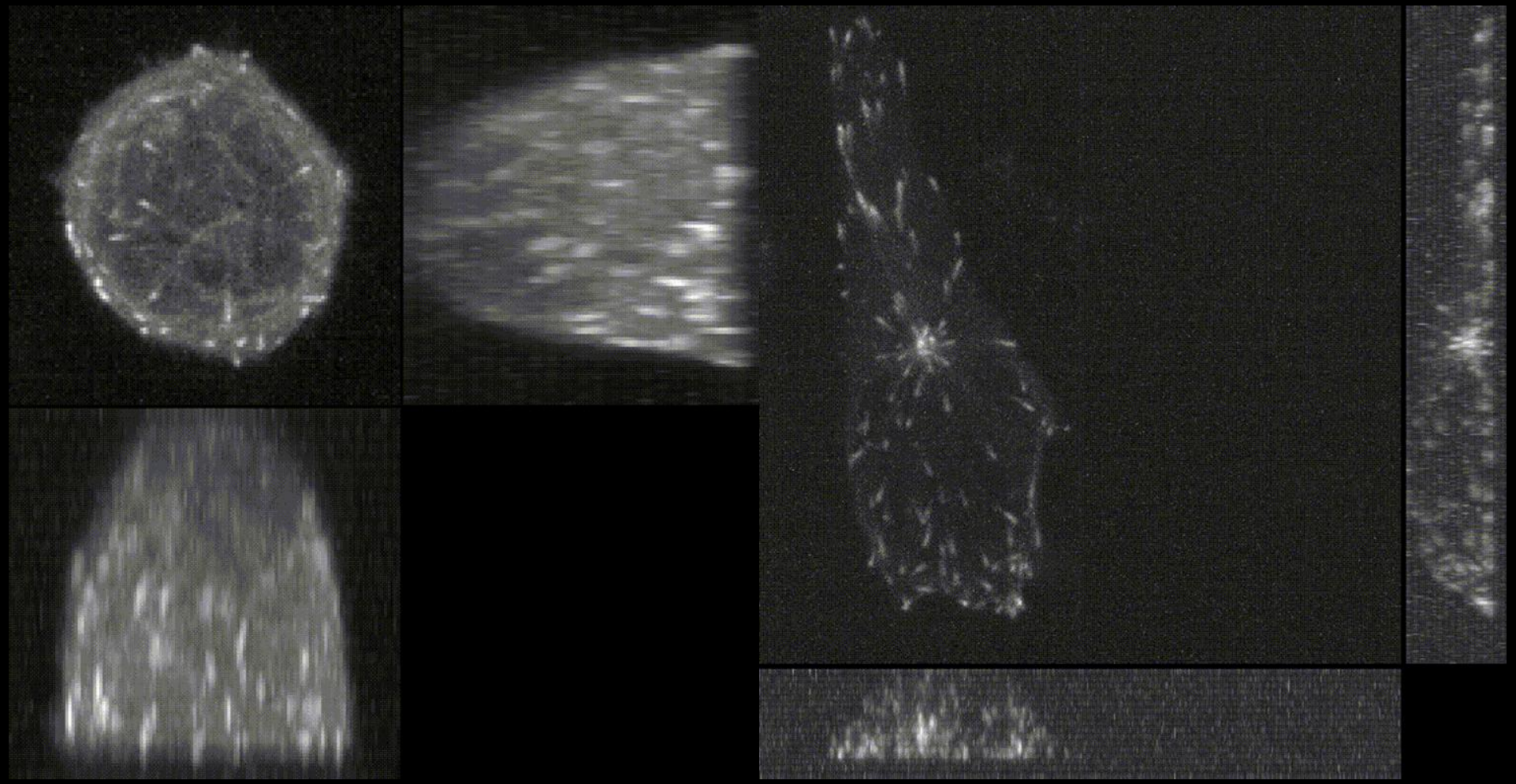
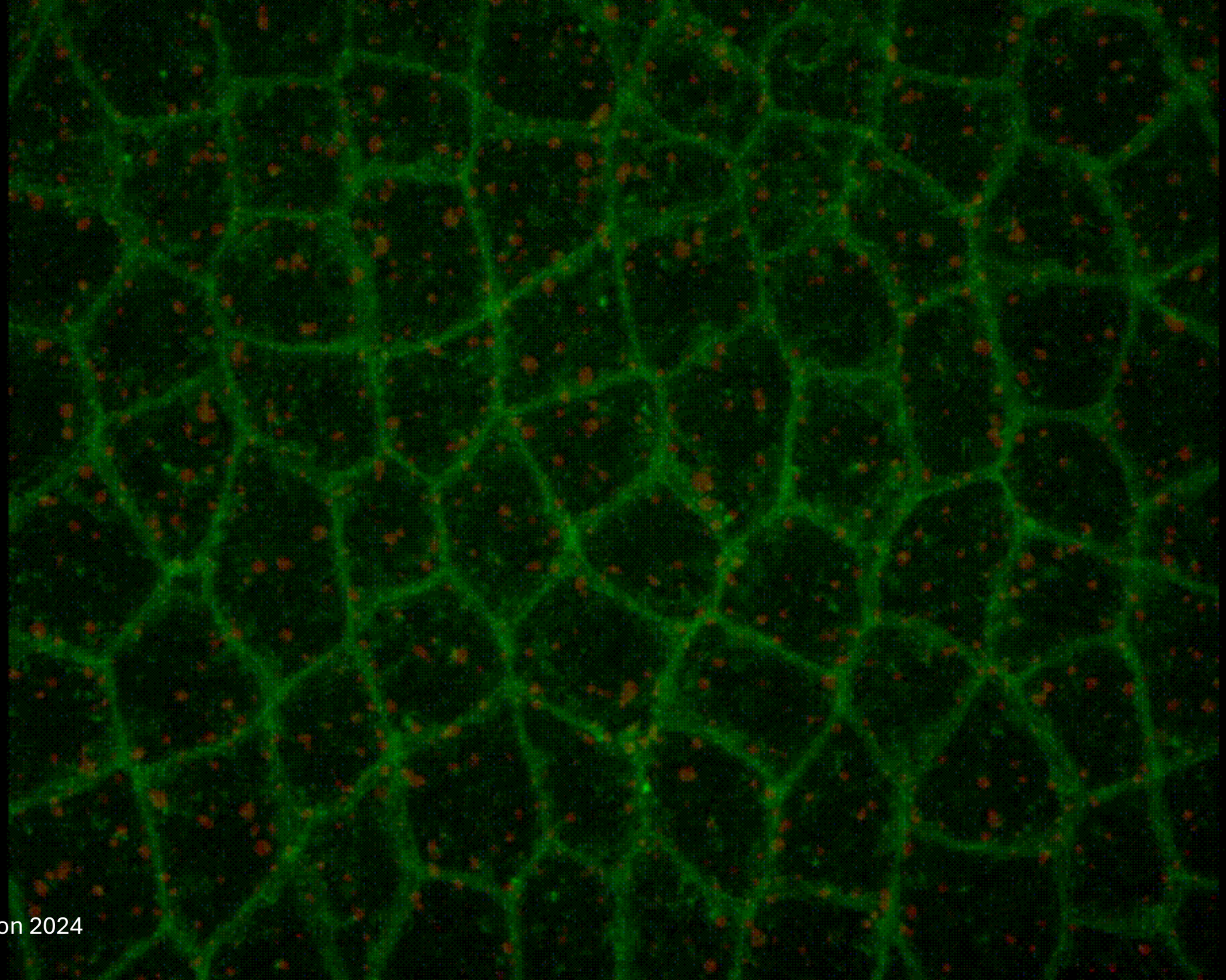
I2M & Inst. Fresnel

Sup: Philippe ROUDOT



G.D.C Ortega,
ETH Zurich, 2023



Team Endotrack,
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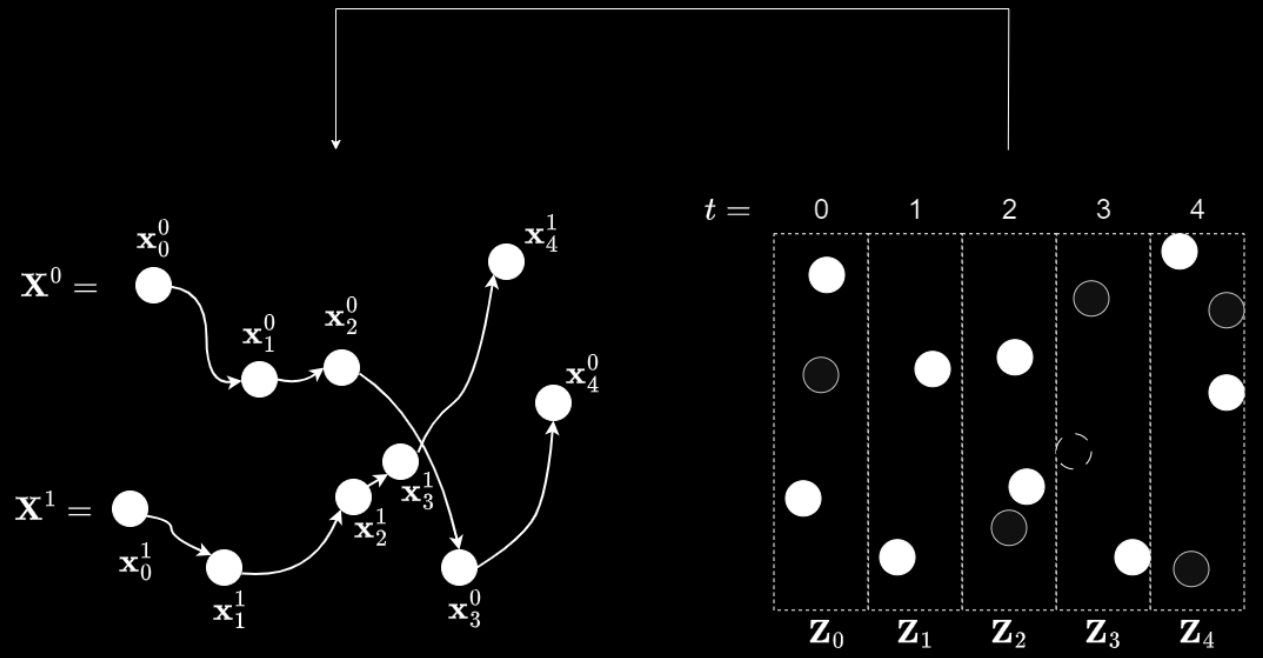
Mapping measurements to states is an inverse problem of data-association



$\mathbf{X} = \{\mathbf{X^P}\}_{p=0:N-1}$

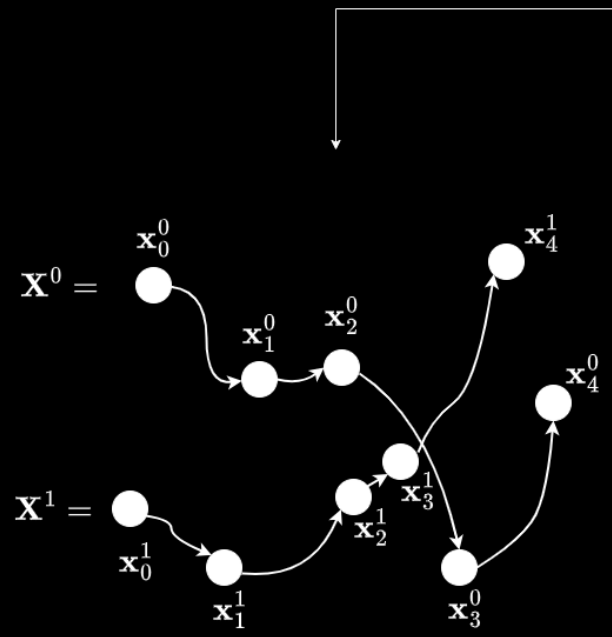
$\mathbf{Z} = \{\mathbf{Z}_t\}_{t=0:T-1}$

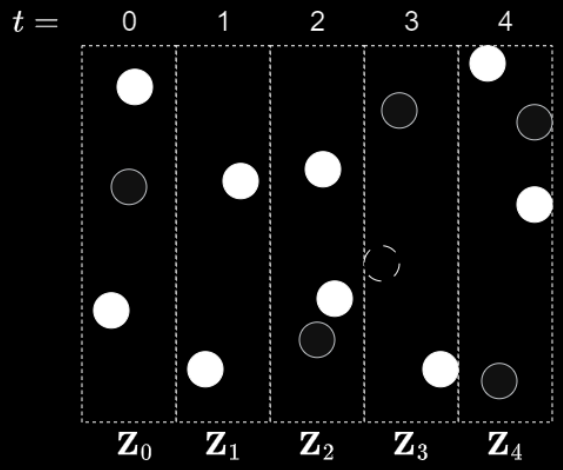
states

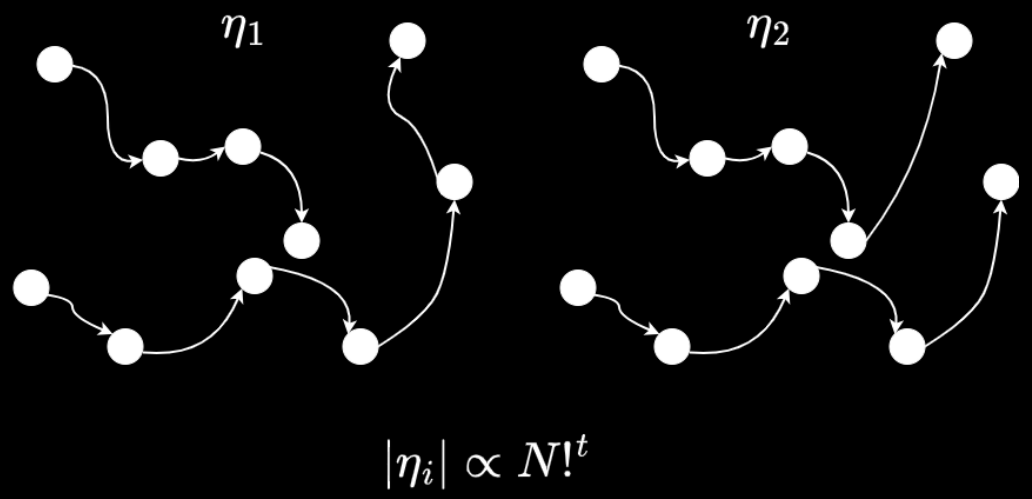
measurements

$$\mathbf{Z} = \mathbf{\Lambda} \cdot \mathbf{X} + \epsilon$$

Data-association is a combinatorially hard problem







$$|\eta_i| \propto N!^t$$

$$\mathbf{X} = \{\mathbf{X}^{\mathbf{p}}\}_{p=0:N-1}$$

$$\mathbf{Z} = \{\mathbf{Z}_t\}_{t=0:T-1}$$

states

measurements

hypotheses

$$\mathbf{Z} = \mathbf{\Lambda} \cdot \mathbf{X} + \epsilon$$

Conventional methods use an iterative estimator as a suboptimal solution

$$p(\mathbf{X}_t|\mathbf{Z}_{1:t}) = p(\mathbf{Z}_t|\mathbf{X}_t) \int p(\mathbf{X}_t|\mathbf{X}_{t-1}) p(\mathbf{X}_{t-1}|\mathbf{Z}_{1:t-1}) d\mathbf{X}$$

association prediction a priori

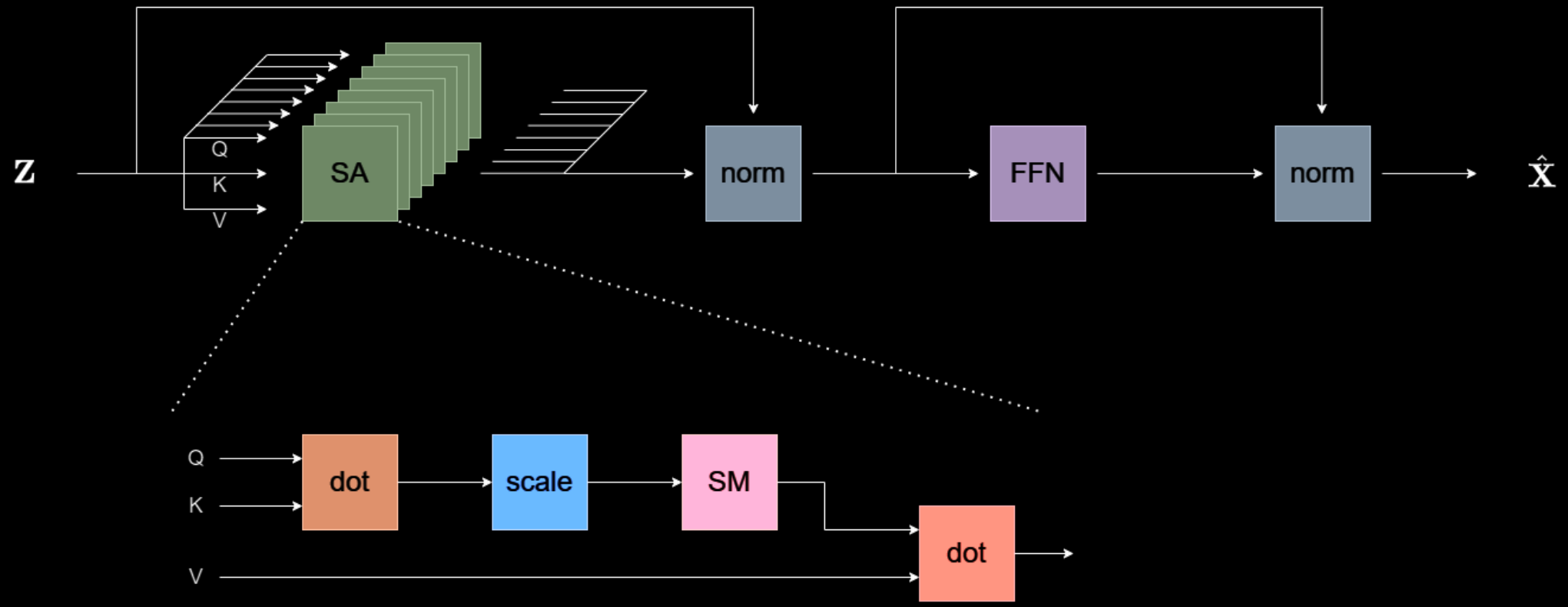
Conventional methods must prematurely prune hypotheses based on priors

$$p(\mathbf{X}_t|\mathbf{Z}_{1:t}) = p(\mathbf{Z}_t|\mathbf{X}_t) \int p(\mathbf{X}_t|\mathbf{X}_{t-1}) p(\mathbf{X}_{t-1}|\mathbf{Z}_{1:t-1}) d\mathbf{X}$$

association prediction a priori

$$= \sum_{\boldsymbol{\eta}_p^t \in \mathbf{H}_t'} p(\mathbf{Z}_t|\mathbf{X}_t, \boldsymbol{\eta}_p^t) p(\boldsymbol{\eta}_p^t|\mathbf{X}_t) \int p(\mathbf{X}_t|\mathbf{X}_{t-1}) p(\mathbf{X}_{t-1}|\mathbf{Z}_{1:t-1}) d\mathbf{X}$$

Attention can be used to make decisions on both states & hypotheses



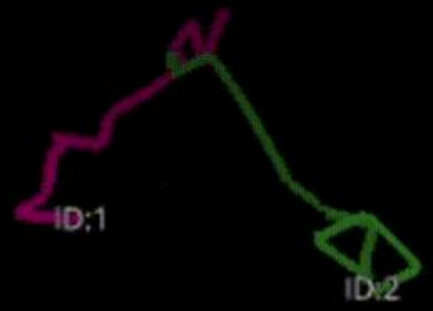
$$\hat{\mathbf{X}} = \text{concat}\left(\text{softmax}\left(\frac{\mathbf{W}_{q,m}\mathbf{Z}_m(\mathbf{W}_{k,m}\mathbf{Z}_m)^T}{\sqrt{d_k}}\right)\mathbf{W}_{v,m}\mathbf{Z}_m, \forall m \leq n_h\right) \cdot \mathbf{W}_l$$

A simple experimental setup for proof-of-concept

$$y^{t,p} = y^{t-1,p} + \varepsilon^{t,p} + \delta^{p}$$

drift

randomness with process noise



$$z^{t,p} = y^{t,p} + \omega^{p}$$

measurement noise

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Similarity between ground-truth and prediction is given by Jaccard coefficient

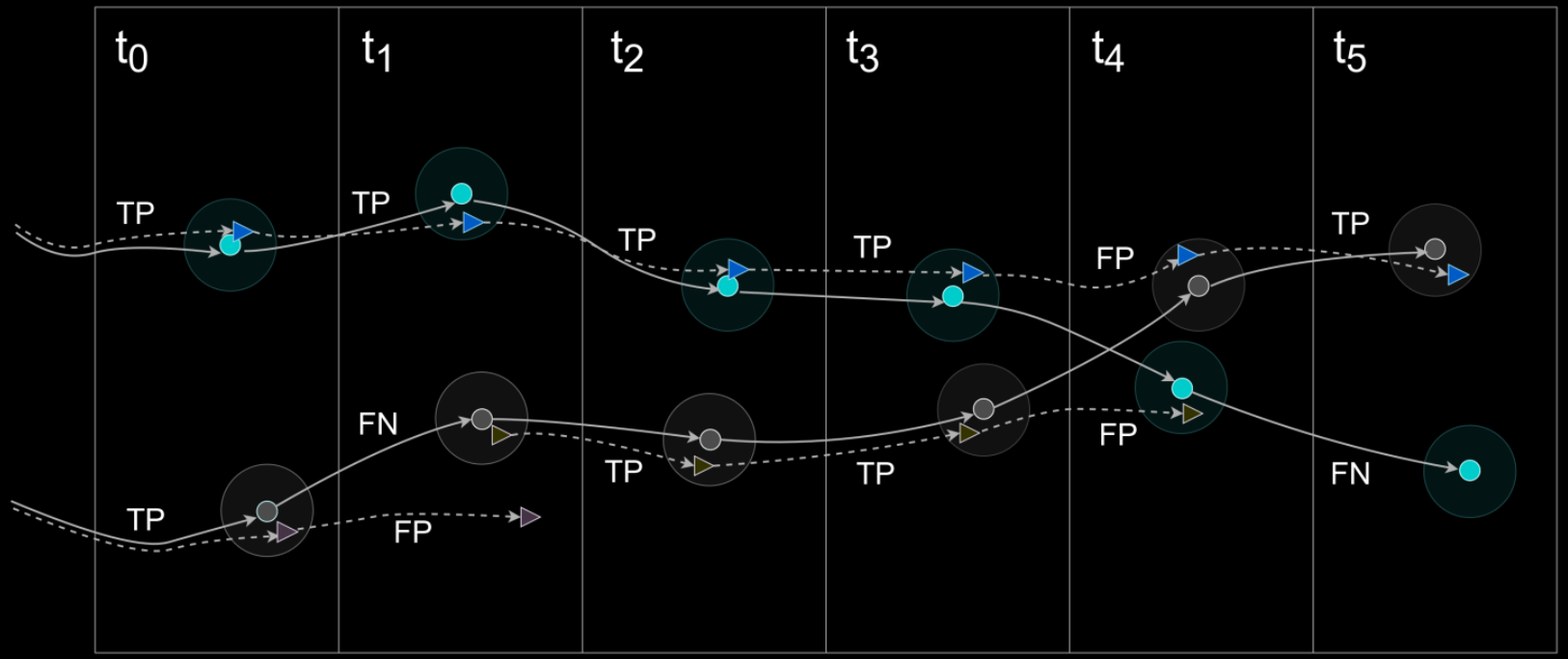
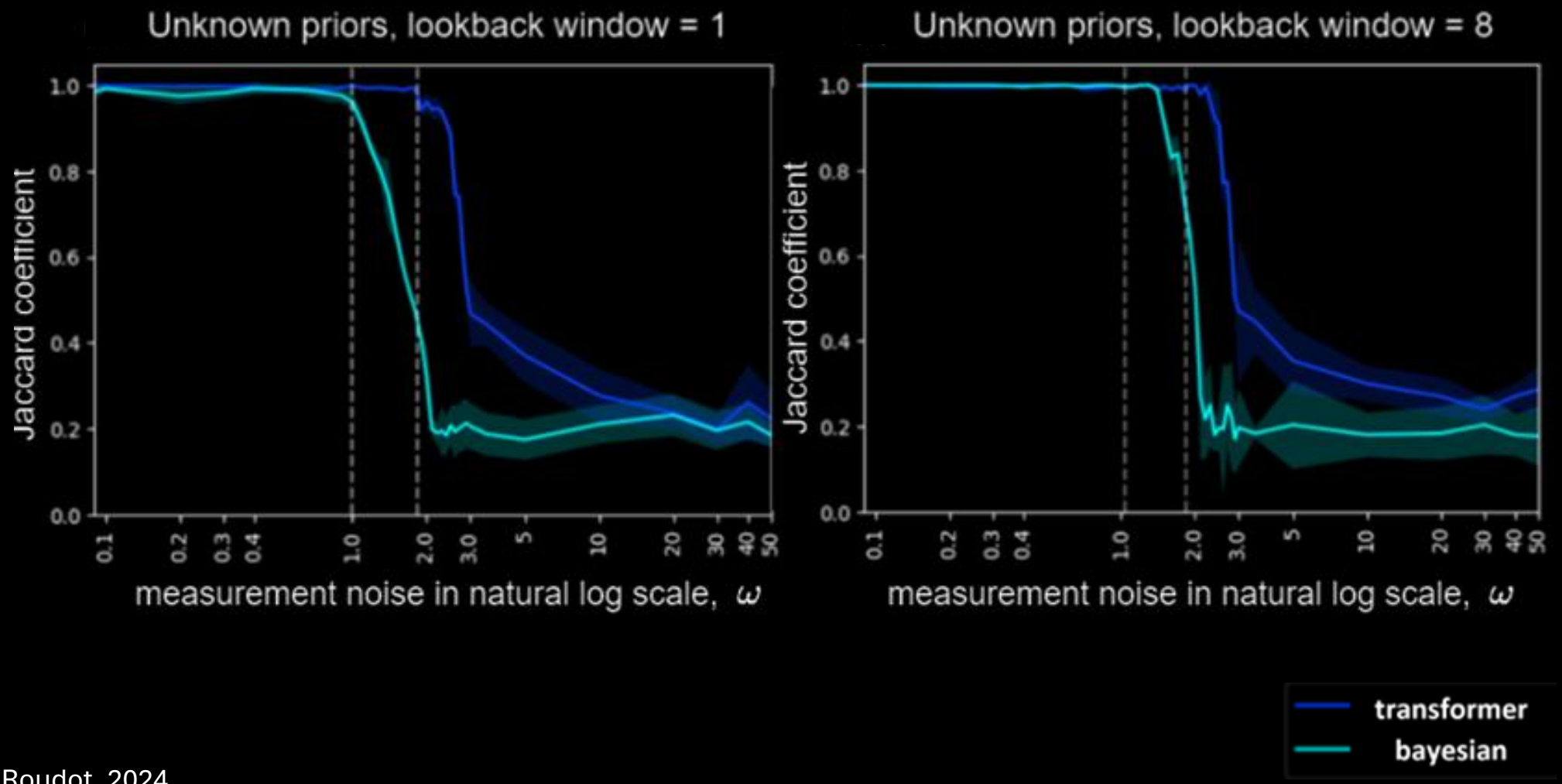


Illustration by Laura Neschen

Attention is robust to increasing noise in long sequences



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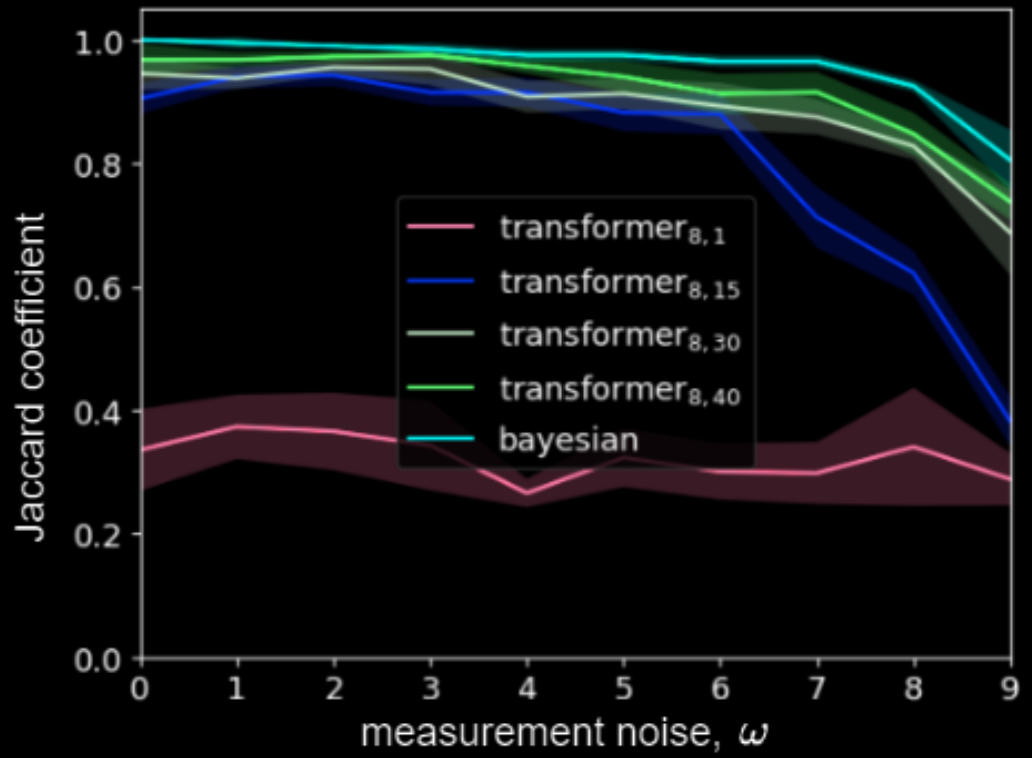
When Bayesian filtering is optimal, attention is suboptimal



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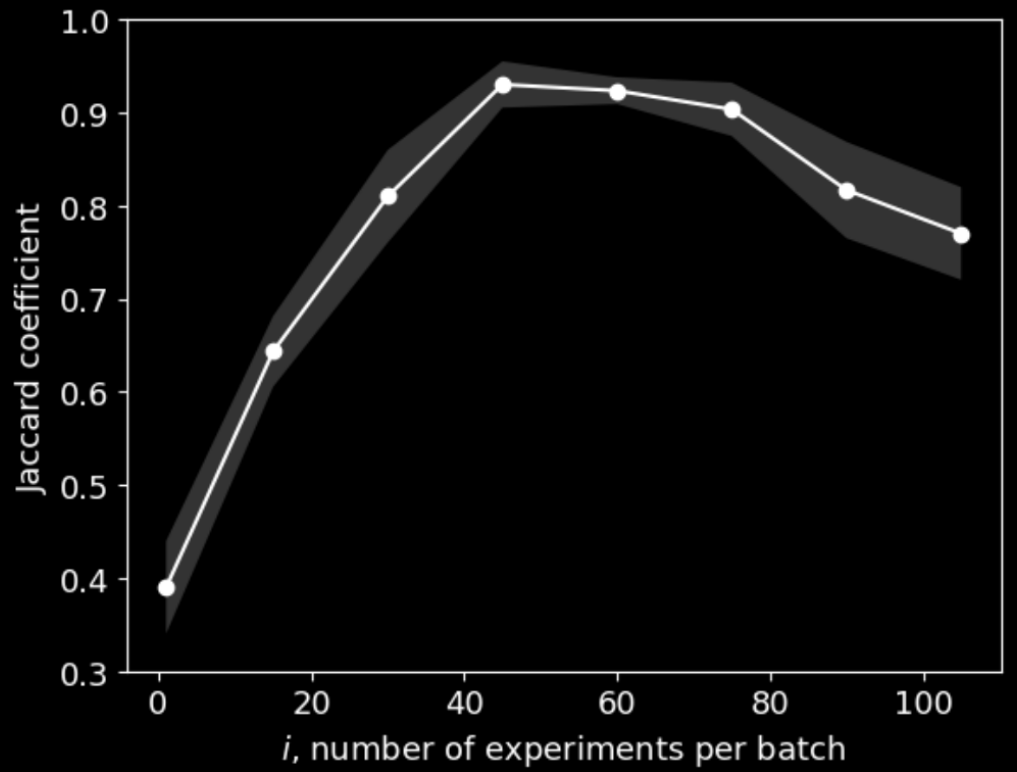

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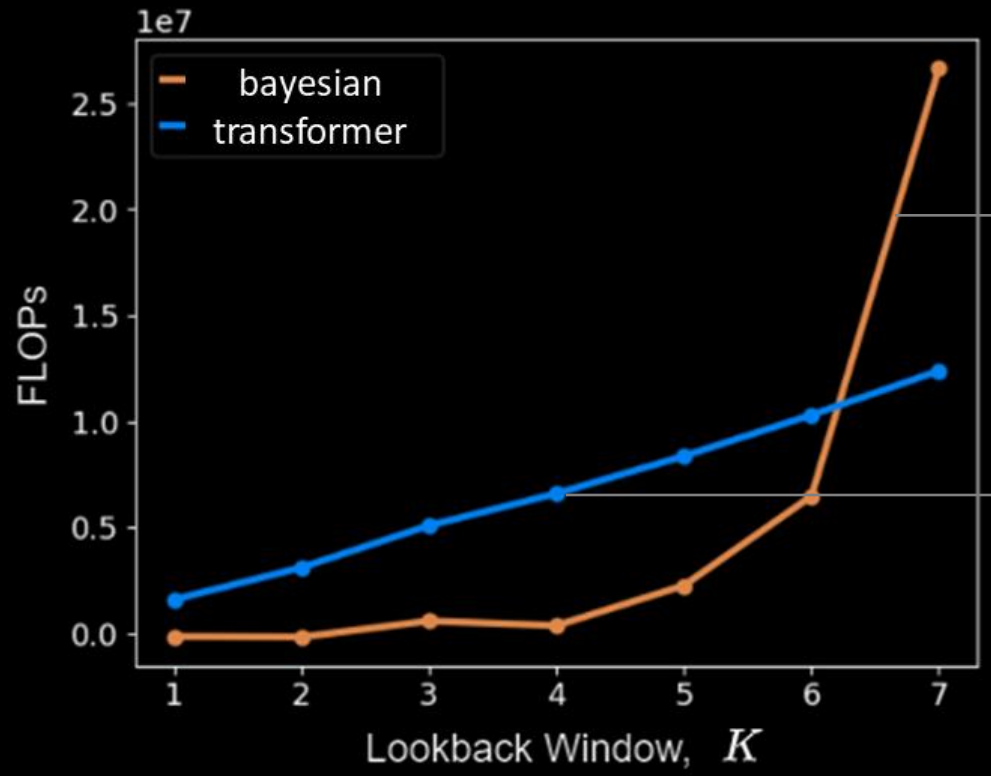
Attention is more efficient when increasing the lookback window



$O(N!^K)$

Reid, 1979

$O(K)$

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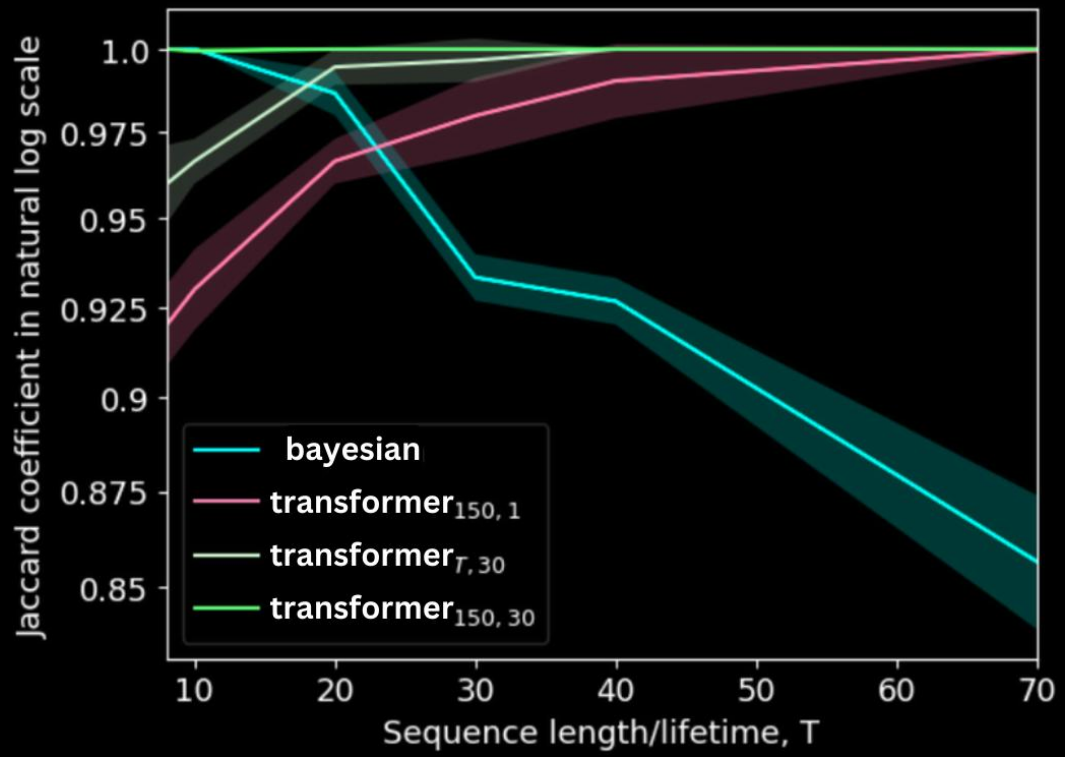
Attention is robust to increasing sequence length



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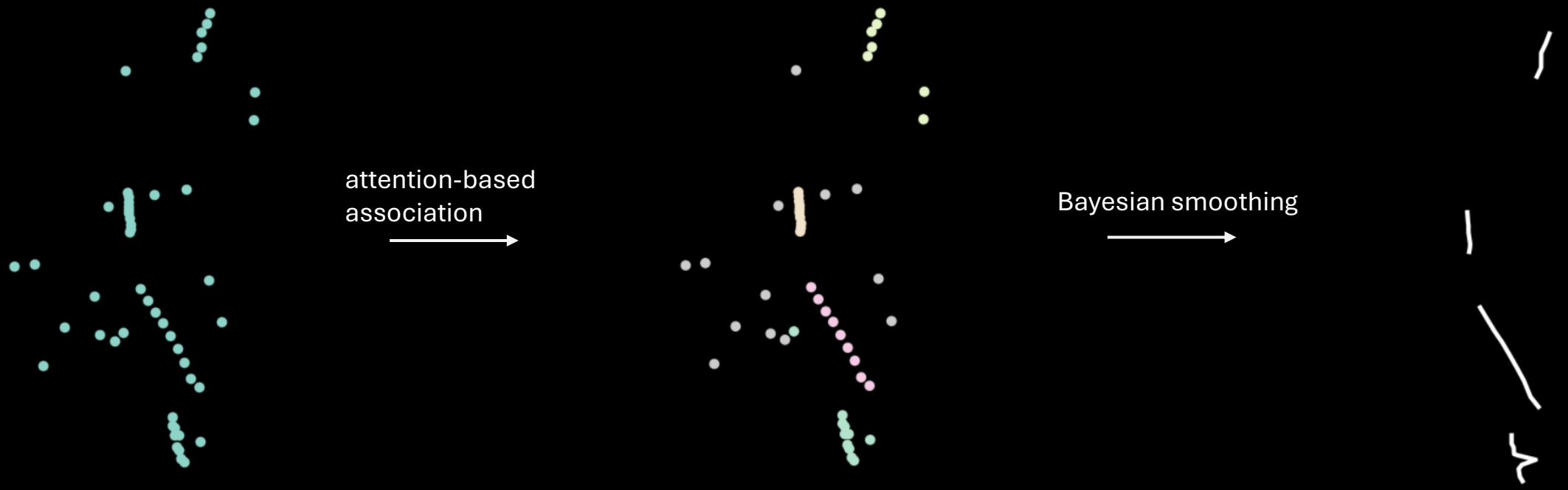
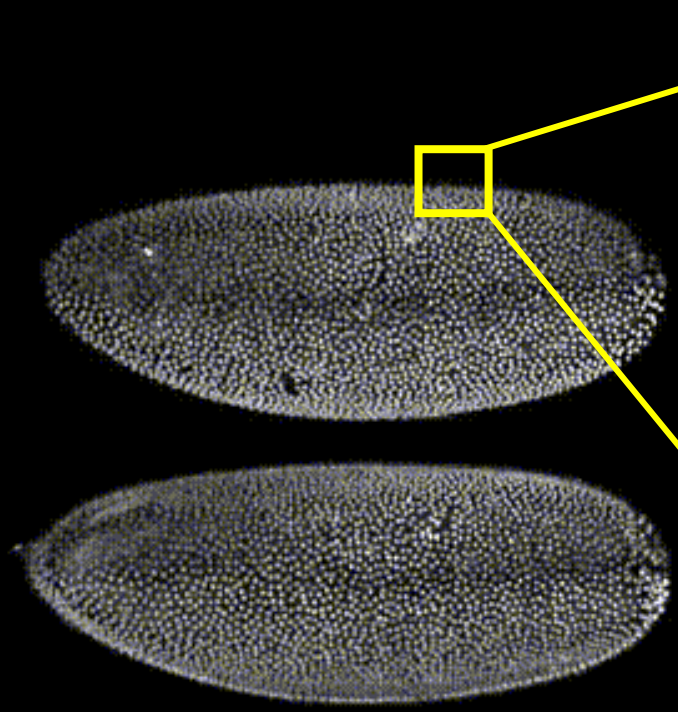
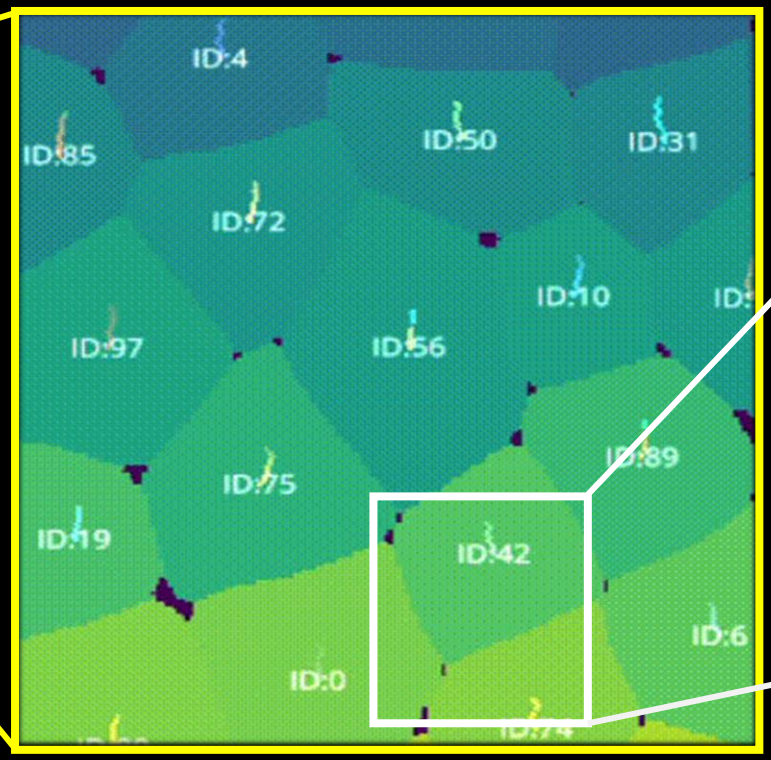
Ongoing work: A frugal tracking strategy that uses attention to build global priors



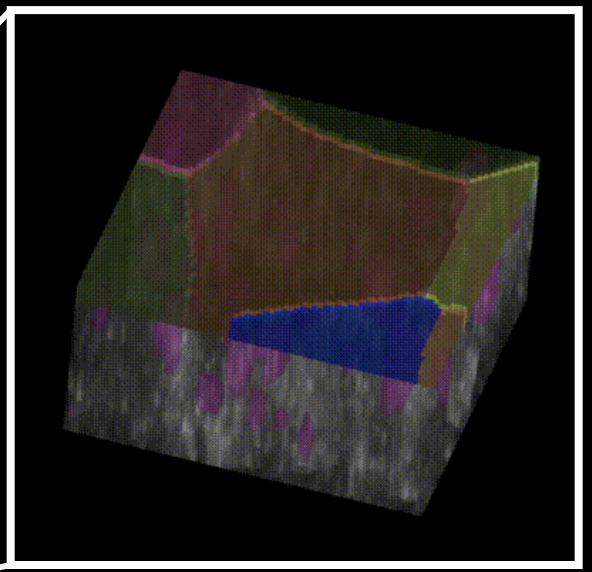
Applications & preliminary results: Stand on a moving cell



Microscopy images
of fruitfly embryo,
C. Collinet, IBDM



Tracking cells using
the Bayesian-Attention
hybrid strategy



Stabilised region of interest
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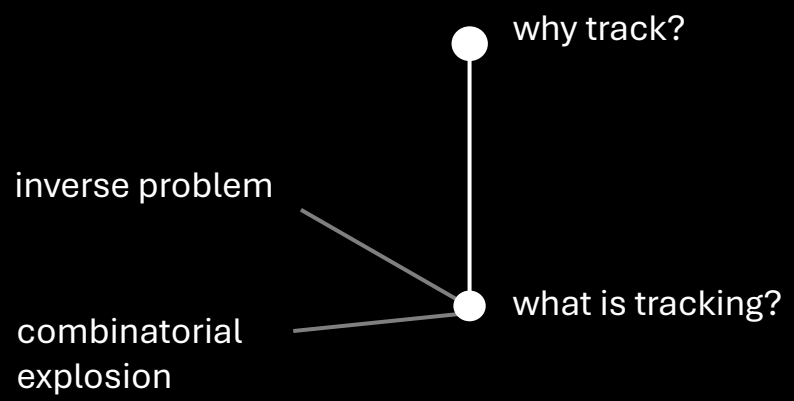
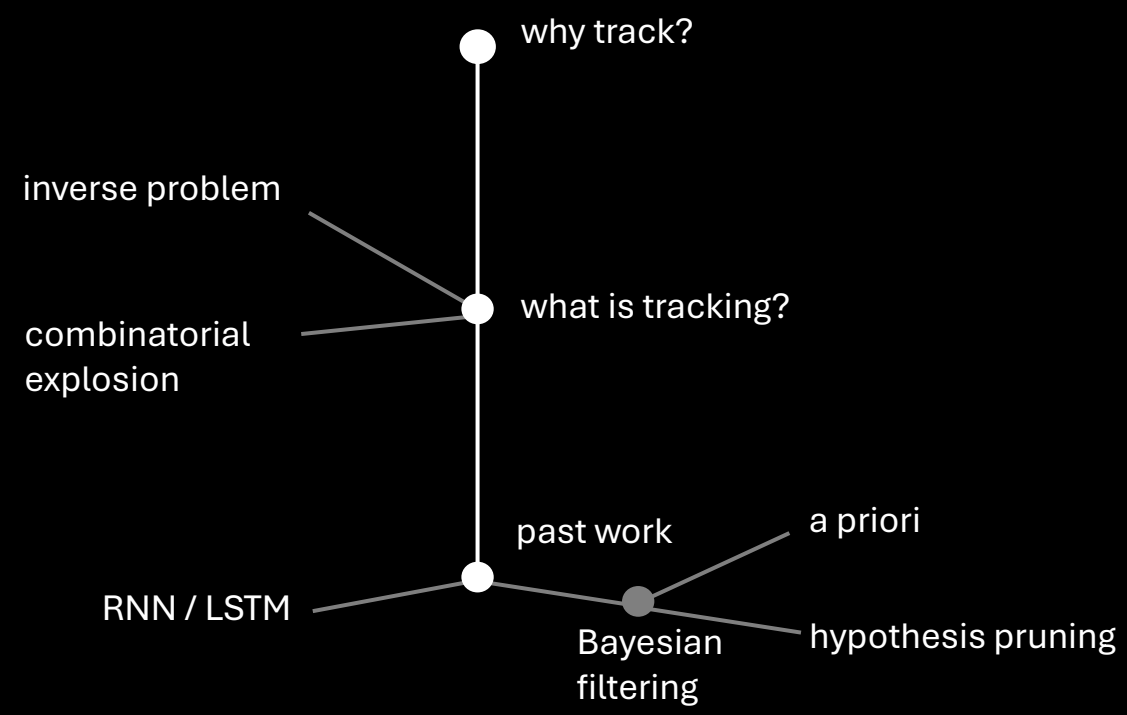
- why track?


why track?
inverse problem
what is tracking?
combinatorial
explosion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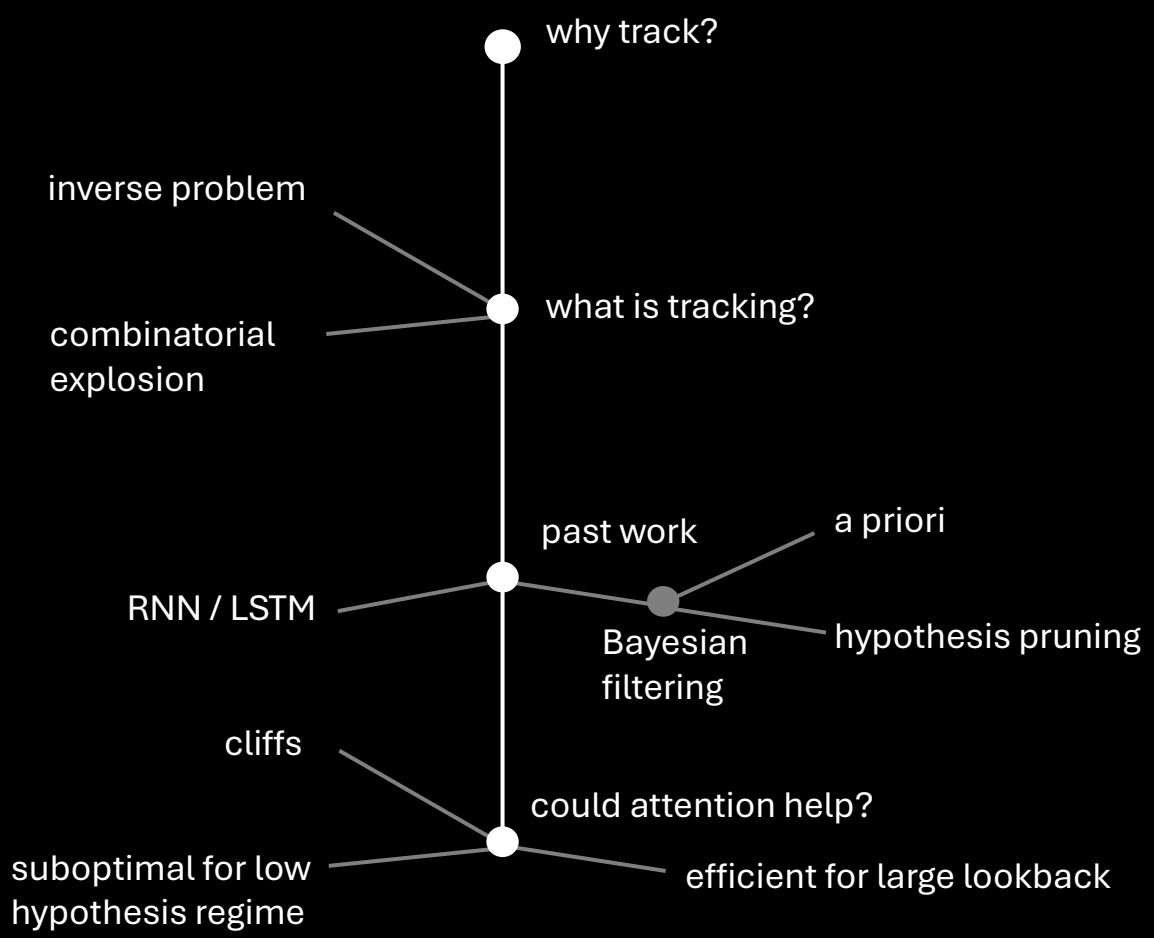
combinatorial explosion
past work
a priori
RNN / LSTM
Bayesian filtering
hypothesis pruning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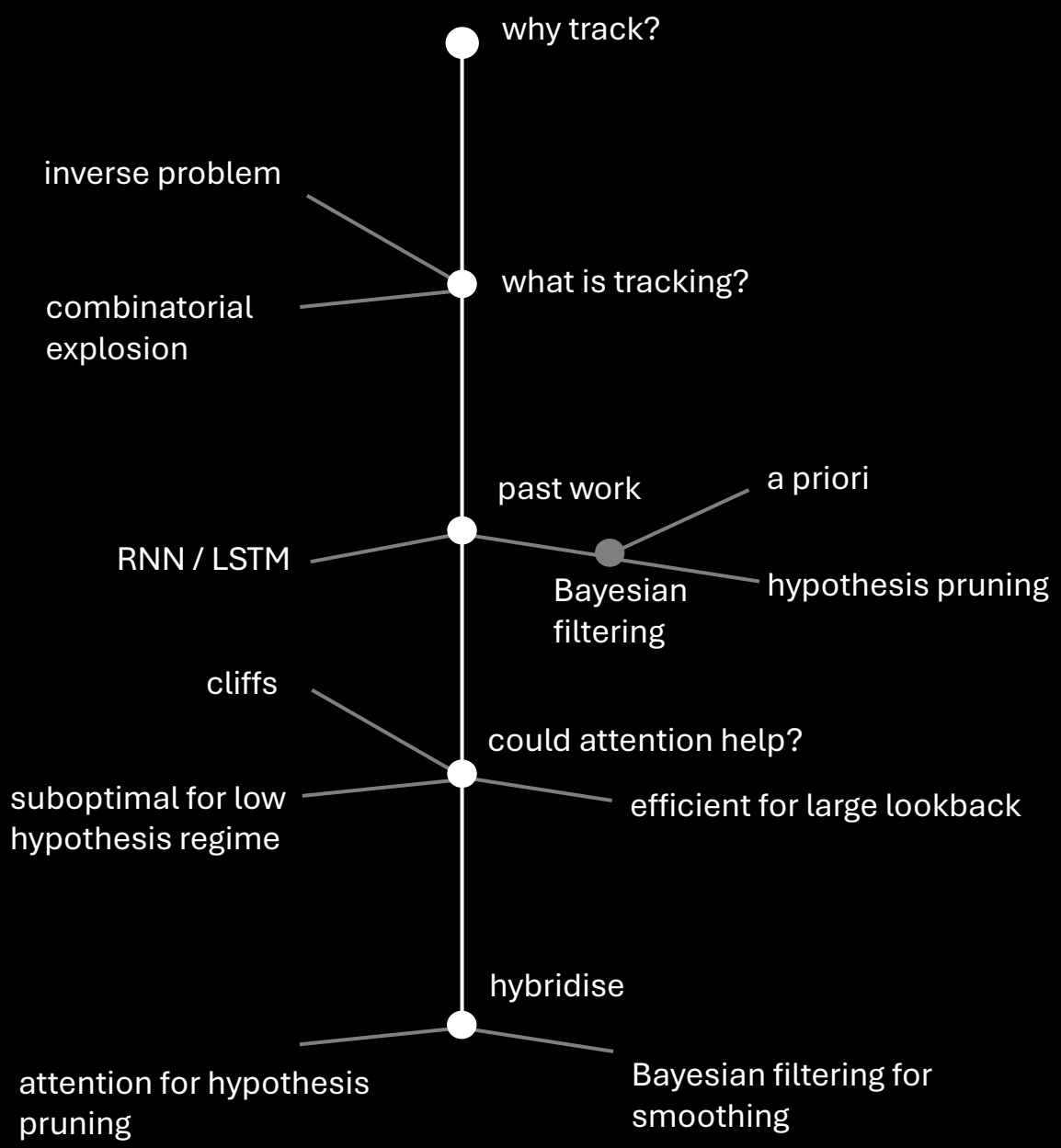
hypothesis pruning
cliffs
could attention help?
suboptimal for low hypothesis regime
efficient for large lookback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hybridise
attention for hypothesis pruning
Bayesian filtering for smoothing

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Thank you for your attention ☺

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